If the universe is at some time dominated by one of these w/ $\rho_{dom} \propto a^{-n}$ then:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} a^{-n} \Rightarrow \frac{da}{dt} = \sqrt{\frac{8\pi G}{3}}\, a^{1-\frac{n}{2}} \Rightarrow a^{\frac{n}{2}-1} da = \sqrt{\frac{8\pi G}{3}}\, dt \Rightarrow a(t) = \left(\frac{n}{2}\sqrt{\frac{8\pi G}{3}}\right)^{\frac{2}{n}} t^{\frac{2}{n}}$$

So for $\rho_{dom} = \begin{cases} \rho_m \propto a^{-3} & \Rightarrow a(t) \propto t^{2/3} \\ \rho_R \propto a^{-4} & \Rightarrow a(t) \propto t^{1/2} \\ \rho_c \propto a^{-2} & \Rightarrow a(t) \propto t \\ \rho_v \propto a^{0} & \Rightarrow a(t) \propto e^{Ht} \end{cases}$ ← have to go back and rederive for this one

$$a^{-1} da = \sqrt{\frac{8\pi G}{3}}\, dt \quad w/ \sqrt{\frac{8\pi G}{3}} = H$$

$$\ln a = \sqrt{\frac{8\pi G}{3}}\, t$$

$$a = e^{Ht}$$

Remember that these results are based on a dominant contribution (so we take $\rho_{non\text{-}dom} \sim 0$). But do we expect dominant contributions? Well since everything scales differently w/ $a(t)$ yes.

- For small a (early times) $\Rightarrow \rho_R$ dominant
- Large a (late times) $\Rightarrow \rho_v$ dominant
- Intermediate a could have ρ_m or ρ_c dominant

Of course we can use observation to help figure out just how much ρ_R, ρ_m, ρ_c we have (more on that next time).

Note: That unless ρ_v dominates at early time (highly unlikely) $a(t \to 0) \to 0$ $\boxed{\text{Big Bang!}}$

Tuesday, March 24, 2015
6:57 PM

Cosmology II Our Univers

Last time we learned that a FRW universe dominated at early times by anything other than vacuum energy must have begun w/ a Big Bang. This is of course a naked singularity, though it does not violate the CCC since it is not the result of collapse.

$H = \frac{\dot{a}}{a}$ (current value H_0 is measurable)

measurable

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2} \qquad a(t),\ \rho(t),\ K$$

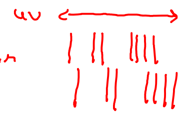
ρ_{tot} (discuss various types below)

Cosmology is in some sense an observational science as opposed to an experimental one. In fact one may argue that much of GR is the same since we cannot "create" any significant sources, though we can "experiment" by observing test masses. Furthermore, cosmology is the study of the time-varying history of our universe. And unlike the criterion for any good experiment, it won't repeat itself.

So what do we observe? Much of our precise knowledge is the byproduct of astrophysics. In this, the detailed study of stellar models including both nuclear and gravitational effects have given us a pretty clear prediction of how certain stars should behave, e.g. their luminosity, size, emission spectra, etc. Upon observing these "standard candles" any distortion from their predicted features actually gives us information on the nontrivial geometry through which their light is moving, i.e. the shape of our universe.

Sunday, April 30, 2017

<u>Emission Spectra</u>: If we predict an emission spectrum for a standard candle, and then observe one that is shifted we can relate the size of the redshift to the relative sizes of the universe between the time of emission (long ago) and observation (now), i.e. $\frac{\omega_0}{\omega_e} = \frac{a_e}{a_0}$



Redshift factor: $z \equiv \frac{\lambda_0 - \lambda_e}{\lambda_e} = \frac{\lambda_0}{\lambda_e} - 1 = \frac{\omega_e}{\omega_0} - 1 = \frac{a_0}{a_e} - 1 \Rightarrow a_e = \frac{a_0}{1+z}$

- We observe $z > 0$ for SC $\Rightarrow a_0 > a_e$, i.e. expansion.

Recall that $H = \frac{\dot{a}}{a} \Rightarrow H_0 = \frac{\dot{a}_0}{a_0}$ $\quad f(t_e \approx t_0) \simeq f(t_0) + (t - t_0) f'(t_0)$

[low redshift]

$$a_e \simeq a_0 + (t_e - t_0)\dot{a}_0$$

$$\frac{a_e}{a_0} \simeq 1 + (t_e - t_0) H_0$$

Then $z \simeq \frac{1}{1 + (t_e - t_0)H_0} - 1 \simeq (t_0 - t_e) H_0$ $\qquad \frac{1}{1-x} \sim 1 + x$

Assuming the light travelled at $c = 1 \Rightarrow t_0 - t_e = d \Rightarrow z = d H_0$

we would like to know this

we can measure this

can we independently measure this?

The notion of "distance" in cosmology is quite complicated. I can think of at least five distinct notions of distance that are useful.

- coordinate distance - this one is useful in computation but not "physical"
- equal time distance to a distant object (how far is it from us <u>now</u>?) Less directly tied to observation.
- observed distance to a distant object (which indicates its distance at the time of light emission that is observed now)
- angular separation - the distance between two distant sources (at approximately the same distance from us)
- angular size of a source - the length across a distant source (note that in contrast to angular separation, this does not expand with time since the object is bound by larger forces)

All of these can be related when we know the full geometry. Fortunately, at small redshift z they all are approximately the same.

Yet another distance that is useful for standard candles is:

<u>Luminosity distance</u>: $L = 4\pi d_L^2 F$

measured flux

"known" luminosity of SC

Knowing L and measuring F let's us calculate $d_L = \sqrt{\frac{L}{4\pi F}}$

For low redshift observations we can use $d = d_L$ and combined w/ z measurements determine a value for H_0:

$H_0 = 70 \pm 10 \frac{\text{km}}{\text{s Mpc}} \Rightarrow d_H = \frac{c}{H_0} = 4.55$ million parsecs $\qquad$ parsec $= 3.086 \times 10^{13}$ km

$t_H = \frac{1}{H_0} = 14.4$ billion yrs

For large redshifts things get much more complicated!

Turning to the other quantities to be determined, define: $\Omega_i \equiv \frac{8\pi G}{3H^2}\rho_i = \frac{\rho_i}{\rho_{crit}}$ (i = M, R, C, V)

where $\rho_{crit} = \frac{3H^2}{8\pi G}$ is the total energy density needed so that $K = 0$, i.e. the universe is spatially flat.

If $\sum_i \Omega_i = 1$ then $K = 0$. $\Leftarrow$

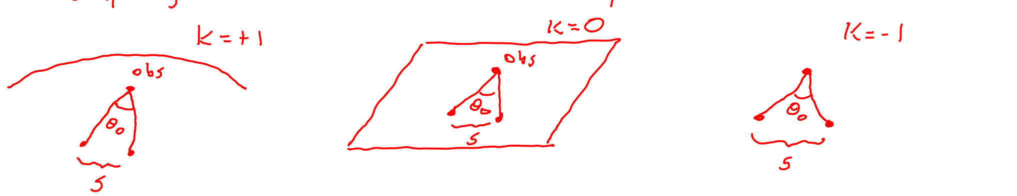
Of course we make measurements at the current time so $\Omega_{i0} = \frac{8\pi G}{3H_0^2}\rho_{i0}$

Ω_{M0}: Look at a cluster and use local gravitational effects to infer mass. Then use density of clusters to extrapolate to large scale.
$\Omega_{M0} \simeq 0.3 \pm 0.1$

Ω_{R0}: Cosmic Microwave Background (CMB) arises from relic photons after last scattering (once e^-, p^+ cooled to form neutral atoms). We measure a thermal distribution w/ $T = 2.73\,°K$ and hence.
$\Omega_{R0} \simeq 10^{-4}$

Recall: $\rho_M \propto a^{-3}$, $\rho_R \propto a^{-4}$ so $\rho_{M0} \gg \rho_{R0}$ makes sense

Ω_{C0}: From our understanding of the CMB, we can predict anisotropies over a length scale s. We then observe the angular size of the anisotropies and comparing to s we can determine the spatial curvature.



$\theta_0 \quad > \quad \theta_0 \quad > \quad \theta_0$

Our observations indicate that $K = 0 \Rightarrow \Omega_{C0} = 0$ and $\Omega_{tot} = 1$

Ω_{V0}: $\Omega_{tot} = \Omega_{M0} + \cancel{\Omega_{R0}}^{\sim 0} + \cancel{\Omega_{C0}}^{0} + \Omega_{V0} = 1 \Rightarrow \Omega_{V0} \simeq 0.7 \pm 0.1$

Thursday, May 4, 2017
8:16 AM

Puzzles in Cosmology

Dark Matter: The result $\Omega_{m0} = 0.3$ is determined from observed gravitational dynamics (by measuring rotational velocities of spiraling galaxies and inferring the mass needed to hold them together).
However we can also take note of how much matter we "see", i.e. how much luminous matter there is. That amount, $\Omega_{b0} \simeq 0.04 \pm 0.02$ is largely due to baryonic matter (protons, neutrons).
The balance of Ω_{m0}, i.e. $\Omega_{d0} \simeq 0.26$

- must be non-luminous (hence "dark")
- most likely not M.A.C.H.O.s (massive compact halo objects, e.g. BHs, white dwarfs neutron stars, etc.)
- most likely non-baryonic (the program of Big Bang Nucleosynthesis is remarkably good at predicting the relative abundances seen today, but also easy to screw up by modifying its assumptions)
- most likely "cold" (since hot would smear out galaxies)
- axions? (CP-violating θ parameter in SM is ~ 0, to explain PQ proposed making it a field w/  but this gives particles!
- sterile neutrinos? not part of the weak interactions
- W.I.M.P.s? possibly superpartners, KK modes

Dark Energy: Dark energy was coined (by analogy w/ DM) to exhibit our lack of complete understanding of what makes up $\Omega_{v0} = 0.7$.
We expect (and can estimate) a contribution from the zero-point energies of quantum fields in the SM (∞-array of SHOs w/ $\frac{1}{2}\hbar\omega$ ground state).
This contribution would act much like a cosmological constant term $\Lambda g_{\mu\nu}$ in Einstein's eqns. More on this in a minute.
But then we have the "coincidence problem" which is that Ω_m dilutes w/ a^{-3}, while Ω_v is constant, a^0. However we observe $\Omega_{m0} \sim \Omega_{v0}$.
Are we just lucky? Enter Mr. Anthropic?!
Actually one method to address this is to introduce a scalar field ϕ into EE with a potential $V(\phi)$ that is shallow:

The Friedmann equation becomes:

$$\underbrace{\ddot{\phi} + 3H\dot{\phi}}_{} + \frac{dV}{d\phi} = 0$$

$3H$ acts like "friction" dampening the evolution of ϕ

Combined w/ a shallow enough potential ($\frac{dV}{d\phi}$ small), we can have $V(\phi)$ be approximately constant but slowly adjusting. However in this case $T_{\mu\nu}$ includes $\underbrace{-V(\phi)\,g_{\mu\nu}}_{\text{just like } \Lambda g_{\mu\nu}!}$

Thursday, May 4, 2017
9:26 AM

Cosmological Constant Problem: With regards to the value of Λ, we can actually get an estimate on its <u>expected</u> value by considering various contributions from particle physics. These come in 2 types.

- Anytime a symmetry is spontaneously broken by a field taking a non-symmetric expectation value driven by some effective potential, e.g.  $]\lambda$, the height of the potential represents the energy released and contributes to Λ. We know that $q\bar{q}$ condensation which breaks chiral symmetry contributes $10^{32}\,eV^4$ in energy density (here we have taken $c = 1 = \hbar$ so $[x] = [t] = [E]^{-1}$), while the electroweak symmetry breaking phase transition due to the Higgs's contributes $10^{44}\,eV^4$.
- We also get a contribution by adding the zero-point energies of the Standard Model fields. Roughly, a field is quantized as an array of harmonic oscillators of varying frequencies over each point of space (the frequency matching the field momentum). But we know that a quantum oscillator has a nonzero ground state of energy $\frac{1}{2}\hbar\omega$. Adding all of these we obviously get ∞, but (as usual in particle physics) we expect something else to take over at large enough scales (beyond M_p), so we cuttoff the momentum integrals at M_p, leaving a contribution to Λ of order $10^{108}\,eV^4$.

What is observed? $\Omega_{v0} = 0.7 \Rightarrow 10^{-12}\,eV^4$ 120 orders of magnitude off!!

Resolving this is one of the more crucial fundamental problems facing particle physics/cosmology.

Friday, May 5, 2017
6:37 AM

Baryon Asymmetry: One immediately puzzling feature of our universe is that it is comprised almost exclusively of matter w/ almost no anti-matter. This, despite that the Standard Model seems to favor nearly equal production of each. Perhaps surprising is that even though this seems like a maximally one sided distribution today, if we trace it back to early times in the universe, it actual only amounts to a difference in matter and anti-matter of one part in a billion. That means that early on the universe was composed of a lot of matter and anti-matter which (as it cooled) annihilated away and what we see now (all matter) is that tiny initial difference.

Believe it or not, getting a small asymmetry is even harder than getting none or a maximal one (which can be achieved for example by imposing certain symmetries).

This is still an open question, but Sakharov at least enumerated three essential conditions for generating an asymmetry if the universe was born w/ equal amounts of matter/anti.

- B-violating processes (changes net number of baryons)
  However a process that increases B could also increase $\overline{B}$.
- CP-violating process that favors B production over $\overline{B}$.
  However thermal equilibrium would still drive $n_B = n_{\overline{B}}$ even if the rates are different.
- Departure from thermal equilibrium

Friday, May 5, 2017
6:48 AM

3 additional puzzles have been more or less addressed by the inflationary models first proposed by Guth.

Flatness problem: $\Omega=1$ (or $k=0$) is actually an unstable fixed point under the growth of a. We currently see $\Omega=1$, so are we just lucky? Of course we could just set $\Omega=1$ and it will stay there, but w/out motivation for doing so this amounts to a fine tuning.

Horizon problem: In a universe of finite age, there can exist regions (at certain times) which have never been in causal contact, i.e. they are separated by a cosmological horizon. At the time the CMB was formed, the universe was large enough to have many causally disconnected regions. Yet the CMB is observed to be remarkably uniform! [Compare w/ ferromagnets]

Relic (or monopole) problem: Whenever a gauge symmetry w/ a $U(1)$ subfactor is spontaneously broken, we would expect the production of at least one topological defect, i.e. magnetic monopole, per causally connected domain. But we observe no monopoles (or vanishingly tiny density).

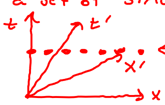
All of these can be addressed by supposing that the universe, in its past, underwent some period of exponential expansion $a \propto e^{\#t}$ (as opposed to power law $a \propto t^n$). Then:

- During inflationary growth $\Omega=1$ actually becomes a <u>stable</u> fixed point and so inflation does the "fine-tuning" for us.
- If inflation occurred <u>before</u> the CMB was formed then everything could have been causally connected prior and now only appear to be (based on power law growth).
- If inflation occurred <u>after</u> monopoles were formed, then their density could be "deflated" $\Rightarrow 0$.

Tuesday, March 24, 2015
6:57 PM

GR in the rearview mirror

We started with special relativity, a framework for doing physics that provided the same value of the speed of light to all observers. This, coupled w/ the general relativity principle that the laws of physics should appear the same to all inertial observers forced us to give up absolute time (everyone agrees on exact time sequencing) and instead adopt a unified spacetime wherein certain generalized rotations (boosts) mix the spatial and time axes, thus allowing a set of simultaneous events in one frame to have spatially dependent time ordering in another:



← simultaneous in x-t, but not in x'-t'

In addition to finding spacetime diagrams like this useful, we also discovered the need to redefine causality from simple time-ordering to a new form in terms of light-cones.

Exploring the relativity principle (form invariance of physics) led to many interesting conclusions

- The Poincaré group (6 Lorentz + 4 translations) or $SO(1,3) \rtimes T^4$ is the relevant symmetry group and all such transformations are specified by <u>constants</u> and relate Cartesian to Cartesian coord. systems.
- All quantities from 3D had to be generalized to 4D including energy (3D scalar) and momentum (3D vector) which effectively unifies them into p^μ
- The 3D→4D program forced us to reckon w/ the general notion of tensors which generalize scalars + vectors
- The construction of densities led us to the conclusion that a 4D energy-momentum density is given by $T^{\mu\nu}$
- We got consistent tools for describing the behavior of massless particles, e.g. $ds^2 = 0$, $p_\mu p^\mu = 0$
- The overarching principle from which much of this flows is that SR is a theory on $\mathbb{M}^4$, a 4D spacetime whose metric in Cartesian coordinates is $(-1, 1, 1, 1)$

Thursday, May 2, 2019
3:29 PM

With special relativity tucked firmly in our back pocket, we then followed Einstein to consider how we might generalize Newtonian gravity to a relativistic context. Instead of following Einstein's logic (which we did, and is interesting in its own right and historic) we will adopt another approach which brings the discussion much more in line with how we understand the other forces.

In hindsight, we know that $\mathbb{M}^4$ is a spacetime in which curvature/gravity is absent. Its symmetry transformations (used to define tensors) are in terms of constant parameters, e.g. $\Lambda = \begin{pmatrix} 1 & & & \\ & \cos\theta & -\sin\theta & \\ & \sin\theta & \cos\theta & \\ & & & 1 \end{pmatrix}$ or $\Lambda = \begin{pmatrix} \gamma & \gamma\beta & & \\ \gamma\beta & \gamma & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$

$T^\mu{}_\nu \to T^{\mu'}{}_{\nu'} = \Lambda^{\mu'}{}_\mu \Lambda^\nu{}_{\nu'} T^\mu{}_\nu$

However we might wish to "generalise" this framework to one that accomodates position dependent (or local) or even arbitrary coordinate transformations. This means 2 things:

- The form of the metric can change, e.g. Cartesian $(-1,1,1,1)$ to polar $(-1,1,r^2,r^2\sin^2\theta)$
- The derivative must be redefined so that $\partial_\mu T^{\lambda\nu} \to \nabla_\mu T^{\lambda\nu}$ is a tensor $\Rightarrow \nabla_\mu = \partial_\mu + \underbrace{\Gamma^{\cdot}_{\cdot\cdot}}$

Christoffel Connection or "Gauge Field"

All of this has been done on $\mathbb{M}^4$ but now accomodates arbitrary coordinates.
The next big step is to not restrict ourselves to $\mathbb{M}^4$, i.e. allow the geometry to become dynamical.
To do so we introduce a gauge field kinetic term identified by $[\nabla_\mu, \nabla_\nu] \Rightarrow R^\lambda{}_{\mu\nu\sigma}$
The resulting theory is now that of a dynamical spacetime and allows arbitrary coordinates, and also inherits simple dynamical principles from $\mathbb{M}^4$, i.e. straight line motion of "free" particles $\to$ geodesic motion in curved geometries

This theory can be applied to describe gravity on any spacetime subject to one critical restriction, i.e. that it be smooth and locally equivalent to $\mathbb{M}^4$ (encoding Einstein's equivalence principle) which means manifolds.

However GR predicts the formation of spacetime geometries which are not smooth manifolds, e.g. collapse to black holes. Though GR breaks down at the singularity, it is fully applicable outside of it which means we can use it to explore the geometry both inside and outside of the horizon which leads to all kinds of bizarre results (maximal extensions) and outstanding problems (BH info. paradox).

And finally, we can throw this tool at the whole Enchilada. To review what just did this week is starting some kind of Zeno's paradox, so I think I will leave it there. Thanks for an awesome semester!! :)